

# APPENDIX H

**THE PUBLISHED PAPER IN 2004:  
THE NATURE OF URBAN  
MORPHOLOGICAL EVOLUTION  
BASED ON CHAOS THEORY AND  
FRACTAL GEOMETRY**

## THE NATURE OF URBAN MORPHOLOGICAL EVOLUTION BASED ON CHAOS THEORY AND FRACTAL GEOMETRY

*Fractal dimension assessment of urban environment at the street Level for the case study of Tehran, Iran*

TOOFAN HAGHANI

Research Department, School of Architecture, University of Central England, Birmingham, UK, B42 2SU  
*Toofan.haghani@uce.ac.uk*

**Abstract.** Chaos and fractal theory allow the conceptualization, evaluation and simulation of complex, irregular, live systems such as cities. This paper intends to highlight some current researches, which examined fractal applications and their links with urban design and analysis. It proposes a new method for physical surveys of the building forms and the street patterns in the north of Tehran, Iran, based on fractal Geometry. It employs a fractal calculator (Fractal software) to facilitate the fractal dimension measurements of Tehran urban environment in order to identify a specific fractal fingerprint, in terms of 'Fractal Neighborhood Identification code' (FNID) at the street level. Then it applies the method to selected streets within the case study for measuring the degree of homogeneity or heterogeneity that they display according to their range of building types. Finally it constructs a platform base for examining more accurately urban forms, and evaluating quantitatively urban change and development.

### 1. Introduction

Towns and cities are complex systems. They are the product of different factors such as politics, economics, technology, fashion, culture, climate, etc. However, until relatively recently they have not generally been treated as complex systems. The planners, designers and builders of settlements treated them as simple predictable systems to be ordered and reduced to its components to facilitate modeling and manipulation.

Batty (1994) wrote 'For generations, architects and planners have attempted to impose a simple, smooth, visual order on cities in the belief that such order counters the disorder and dysfunction which cities reveal when they develop 'naturally'. Klinger and Salingaros (1999) wrote the greatest creation of humanity –their buildings, cities, artworks, or artifacts- are neither simple nor random, but have a high degree of organized complexity. Chaos and fractal theory help us to conceptualize, evaluate and simulate complex, irregular systems like those exist within cities.

The main proposition of this paper is that cities' growth implies a systematic order at each level of their hierarchy. In other words, contained within the growth process are codes, which determine how the organization of these basic units of urban development might repeat their form and function in many scales where subsequently result a unique geometric pattern for each part of a city. The method proposes here, is an attempt to evaluate quantitatively city patterns by measuring mathematically their fractal dimensions and by transferring their geometric forms to the numerical codes extracted from existing fractal pattern of them. The term 'FNID', *Fractal Neighborhood Identification code*, will be defined in this paper, as a kind of fingerprint for each part of city, by which mathematically one could measure the degree of complexity that each part demonstrates at local neighborhood level of Tehran's streets. In Section 2.1 and 2.2 a brief definition for fractal geometry and chaotic systems will explain how these new theories has shifted our conventional views towards more realistic understanding of city form and function.

**Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium**  
A Public Forum of Asian Urban Design

---

Furthermore it explains the ability of this method to explore other city's physical features such as plans, elevations, skylines... and the possible link between their visual appearance and their fractal dimension by which it might be typified. Fractal dimension assessments of environment, also assists urban designers to test their design products and evaluating the level of harmony achieved by a set of building combination of different type within an urban realm.

## **2. Theoretical Background:**

A number of authors such as Lynch (1961, 1981), Alexander (1987, 2000) have attempted to recognize the real world complexity of the cities. Lynch (1961) investigated the psychological complexity of the city in terms of how people navigate their way around a place. Alexander (1987) sought to discover, through the development of a "pattern language" combining urban elements, the quality without a name that makes a place special. Bovill (1996) have developed a design method that attempts to deal with the complexity of a place in terms of its function, appearance and structure. Each of these approaches has the common characteristics of trying to deal with urban places as combination of elements e.g. buildings, spaces, natural features, meaning and symbols.

More recently Authors Batty (1991, 1994, 1996, 1999), Bovill (1996), Jencks (1997) and Salingaros (1999, 2003) and Cooper (2000, 2003) recognizing the complex nature of the city, have suggested the emerging new sciences of Chaos and Fractals, as means of dealing with complexity in planning and design. Many of these attempts are now concentrating on city simulations (e.g. recent work in UCL, London) and developing Cellular Automata tools; It is based on the concept that, city evolution at large scale is traceable by following the sequence of changes occur in small scale units (cells) within it (Batty & Xie, 1994). Therefore since the last two decades, there have been a number of attempts, connecting micro scales to the macros, however, some followed by the notion of finding a kind of DNA signature for cities (Webster, 1995). These together with Cooper's work on fractal analysis of Oxford streets directed my research towards exploring fractal dimensions within the urban realm, and specifying a kind of fingerprint for each urban space in terms of 'FNID'. It provides an appropriate tool for controlling the large scale changes in urban growth by controlling the changes occur in small scale units at local neighborhood level.

### **2.1. CHAOS THEORY**

Both chaos and fractals describe complex systems, chaos describing the processes of changes affecting a system while fractals illustrating the resultant patterns. Chaos might be defined as the unpredictable behavior of nonlinear, complex and dynamical systems. Grace (1991) wrote "*chaos*" is about the study of nonlinear dynamic systems and deals with irregular and unpredictable behavior rather than trying to reduce complex systems to linear cause and effect relationships. In addition to "Unpredictability", Glick (1987) listed "Emergence", "Self organization", 'Adaptability' and "Irreducibility", as some of other characteristics of nonlinear chaotic systems. All these can arguably be observed in the city. In another words, urban systems manifest the characteristics of nonlinear chaotic systems, they are argued to have a self organizing nature; they can emerge, organize and evolve, without a preconceived master plan, yet across both time and space.

One the main characteristic of such a system is irreducibility. A non-linear system can not be reduced to its component parts; in this sense the whole is more than the sum of its part. Jan Walleczek (2000) argued the concept that the dynamical interactions between dependence elements at a local microscopic level influence and influenced by the emergent global structure at the macroscopic level. Through the continuing interactions interplay between micro and macro processes the emergent; self-organizing structure is stabilized and actively maintained. In a city it means that it is impossible to take the plans of a single house and from them deduce the totality of the city. The position of that house and its subsequent neighbors will be subject to a range of many agents in making decisions about the location. However, the way we controls the changes occur in small scale units at local neighborhood level, is an effective way to control the large scale urban changes. As Batty and Longley (1994) believe cities in general, evolving and changing according to their local rules and conditions which manifest more global order across many scales and times.

### **2.2. FRACTAL GEOMETRY**

### Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium A Public Forum of Asian Urban Design

A mathematician, Benoit Mandelbrot, first suggested the term “fractal” in 1977, he mentioned that ‘a fractal is a shape made of parts similar to the whole in some way’ (Feder, 1988, p11). In another words there is self-similarity over deferent scales of a fractal object. Mandelbrot (1982) argued that many of *irregular* and *fragmented* patterns around us in the nature could be described by fractal geometry. It helps us to study those form that Euclid leaves aside as being formless or morphologically amorphous.

According to Barnsley (1993) classical geometry (Euclidian geometry) provides a first approximation to the structure of physical objects; this is the language we use to communicate the designs of technological products and, very approximately, the forms of natural creations. Fractal geometry is an extension of classical geometry. It can be used to make precise models of physical structures from ferns to galaxies. Fractal geometry is a new language. Once you can speak it, you can describe the shape of a natural phenomenon.

In the field of urban planning and design, the discovery of fractal geometry also engendered a shift between the old view - that sees cities as simple, ordered structured, expressible by smooth lines and shapes, which describe their overall morphology and the disposition of their elements - toward a view that cities are complex organisms, evolving and changing according to local rules and conditions, which manifest more global order across many scales and times. Batty and Longley (1994) believe not only naturally growing cities but changes in forms and functions of planned cities over time, are in a way that make them both ideal candidates for application of fractal geometry.

### 3. Fractal analysis of urban morphology

In city ecosystems, the pattern of pedestrian and traffic movement, neighborhood decline, renewal, consolidation and transition will be constrained or influenced by morphological elements. Edges, boundaries, regions, coarseness, fragmentation, homogeneity or heterogeneity of neighborhood patterns are the most important features in both Landscape and urban ecology. According to the recent research, the urban feature defined by its physical elements, better represent city identity than the one defined by its underlying social elements. For example Webster (1995) argued urban environmental categories will be defined by housing density better than population density. The reason should be obvious; population density will vary to some degree between neighborhoods of the same housing density.

Fractal analysis of urban morphological elements offers a base to classify facial pattern recognition the same sort of digital technology fingerprint. It provides urban analyst a way in order to uniquely identify a particular code for an urban form or pattern. It is similar to DNA code or a fingerprint for a forensic scientist who wants to identify its owner. In the case of city, the configuration of shapes and structures will produce that morphological fingerprint.

#### 3.1. THE METHODOLOGY

As illustrated in diagrams 1 and 2, the suggested method will substantially be able to approach the objectives addressed in this paper.

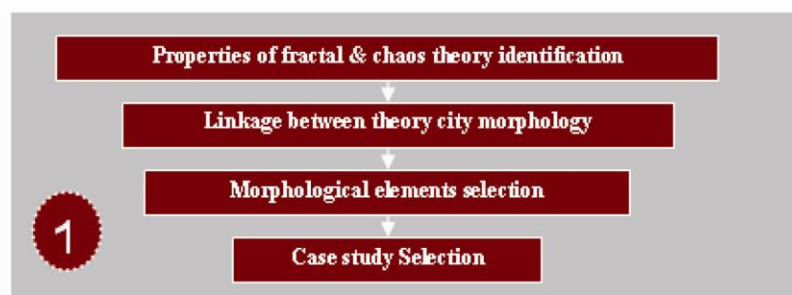


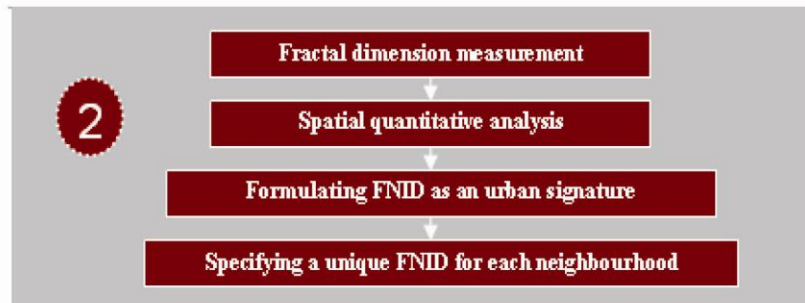
Diagram 1: Theoretical stage and case study preparation

At the theoretical stage the properties of fractal and chaos theory and their possible links with urban morphology has been reviewed. It identifies those elements of the city that lend themselves to the measurement and evaluation by



**Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium**  
A Public Forum of Asian Urban Design

employing fractal analysis software. From the information collected from this part, three different building set were selected to be tested empirically at stage II.

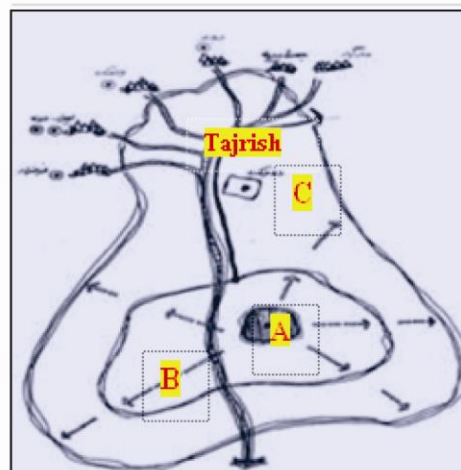


*Diagram 2: Empirical stage and data analysis*

At empirical stage the quantitative data collected by calculating the fractal dimension of urban forms mainly based on available air maps and street panoramic photos. Several streets of Tajrish, a district in the north of Tehran, were selected where the contradiction between traditional organic change and fast urban developments provides an appropriate case for comparison and further analysis. A morphological urban pattern surveyed, assessing fractal dimension of different urban spaces at local street level in order to specify a unique fractal signature (FNID) for each neighborhood unit. The image based morphological analysis offers a language for articulating above idea, a range of precise measurements was undertaken to record distinctive forms in order to parameterize those measures. Image texture measures will present fine discriminations of city features that are defined in terms of built or natural environmental elements.

### 3.2. TEHRAN AS A CASE STUDY:

The morphology of Tehran, the capital of Iran, is the product two different pattern of growth, on one hand a fast and huge expansion of the former city towards its suburbs and on the other hand a gradual organic growth of the villages which were around the city in the past but they are now inside the city (Figure 1).



*Figure 1: the sketch shows Tehran expansion in 20<sup>th</sup> century.*  
A) City boundary in 1890 B) City boundary in 1940  
C) City boundary in 2002

The City of Tehran has experienced the destruction of its past visual order and harmony by new and modern developments during the last century that caused this city to lose their pre-modern identity. The modern pattern of the new constructions is diametrically opposed to old patterns evolved during gradual organic process. Tajrish located in the north of Tehran is a good case manifested the contradiction between both patterns of growth.

**Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium**  
A Public Forum of Asian Urban Design

As illustrated in figure 2 and 3, a similar pattern originated from previous organic growth, obviously exists in the urban fabric in Tajrish repeated at different scale and size. This similar unit could be considered as a cell generator, creating the site morphological pattern. As shown in figure 3, this proportional structured pattern was constructed by a self-similar semi-square shape, repeated at different scale and size in this area. Figure 4 abstractly highlights the existence of the underlying geometrical order within neighborhood units, demonstrating fractal characteristics of urban structure in Tajrish. This preliminary evidence reveals the potentiality of chaos theory and fractal geometry application in urban spatial analysis.



Figure2: Map of Tajrish located in the north of Tehran



Figure 3: Similar pattern originated from organic urban growth exists at different scale and size of Tajrish urban structure.



Figure 4: Fractal pattern distribution as a preliminary evidence of Fractal characteristics of urban structure in Tajrish.

**Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium**  
A Public Forum of Asian Urban Design

### 3.3. FRACTAL CALCULATION

There are several calculation methods for measuring the fractal dimension of the environment; all of them often follow a simple logarithmic equation based on a power law relationship between the numbers of elements exist in different scale of an object.

$$D = \log N \times (\log S^{-1})^{-1}$$

*D: Fractal Dimension, N: Number of elements countable at specific scale, S: Respective Scale of an object*

A Fractal software employed here facilitates the calculation. The data presented in this paper is mainly based on the box counting method and Richardson walk method. These two Methods among other ways of fractal calculation have been fully explained by Mandelbrot (1982), Bovill (1996), Kaye (1889), Cooper (2000) ...

Batty and Longley (1994) applied fractal analysis for measuring the level of complexity exists in the boundary of Cardiff and compare it with some other cities around the world. Table 1 contains some their result.

*Table 1: Fractal dimension of some cities' boundary*

City Name	Year	Fractal Dimension
Tokyo	1960	1.312
New York	1960	1.710
Paris	1981	1.66
Cardiff	1981	1.586
London	1962	1.774

It was examined for the case of Tehran and the result has been summarized in Table 2.

*Table 2: Fractal dimension of Tehran boundary from 1820 to 1992*

City Name	Year	Fractal Dimension
Tehran	1820	1.322
Tehran	1860	1.435
Tehran	1900	1.495
Tehran	1914	1.553
Tehran	1939	1.412
Tehran	1962	1.575
Tehran	1970	1.625
Tehran	1992	1.762

This method could be applied for other urban features such as, city networks, skylines, density of built up area, the degree of open space, Landmark distribution, etc. Any one of these urban elements in combination with the others will assist us to develop the fingerprint idea.

### 3.4. FRACTAL NEIGHBORHOOD IDENTIFICATION CODE (FNID)

Since the new view observed cities as a live organism, some of recent urban scientists have made an attempt to identify a kind of DNA or textural signatures for urban neighborhoods (Webster, 1995). This paper suggests a method to develop the fingerprint idea in term of FNID by which every neighborhood might be morphologically typified. One of the important morphological features that have been addressed here is the degree of building fragmentation by comparing built up areas to open space within urban structures.

Benoit software, a fractal calculator, was employed in this research to facilitate the fractal measurement of urban environments. The inputs are the BMP image format of the maps of the case study in different scales and the soft ware counts the number pixels or the elements out that image. Then a sequence of logarithmic calculations will define the



**Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium**  
A Public Forum of Asian Urban Design

fractal dimensions at different scale. Figure 5, is a plot the number of occupied pixels, as representatives of built up area, and table 3 contains  $F(b)$ , Fractal dimension by box counting method, resulted from each time calculation.

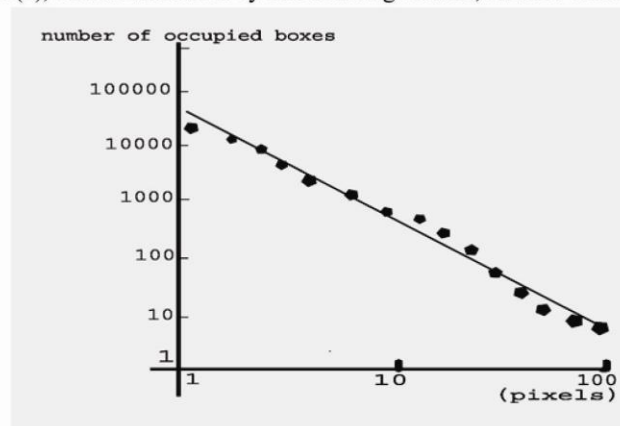


Figure 5: Number of occupied built up area in Tehran map calculated in box counting method

Table 3: Estimating the fractal dimension of Tehran by fixing the scale and varying the size

Box size (pixels)	100	75	50	35	20	10	5	1
$F(b)$	1.484	1.592	1.701	1.769	1.791	1.812	1.693	1.652

As shown in above table the  $F(b)$  is changed when the measurement employs smaller box units (varying the size). The software calculates the average of these as the outcome which in this case would be 1.691 for whole Tehran. In the same way we are able to measure  $F(b)$  for each district of Tehran at different scale. Figure 7 demonstrates the result when the scale is changed (varying the scale).

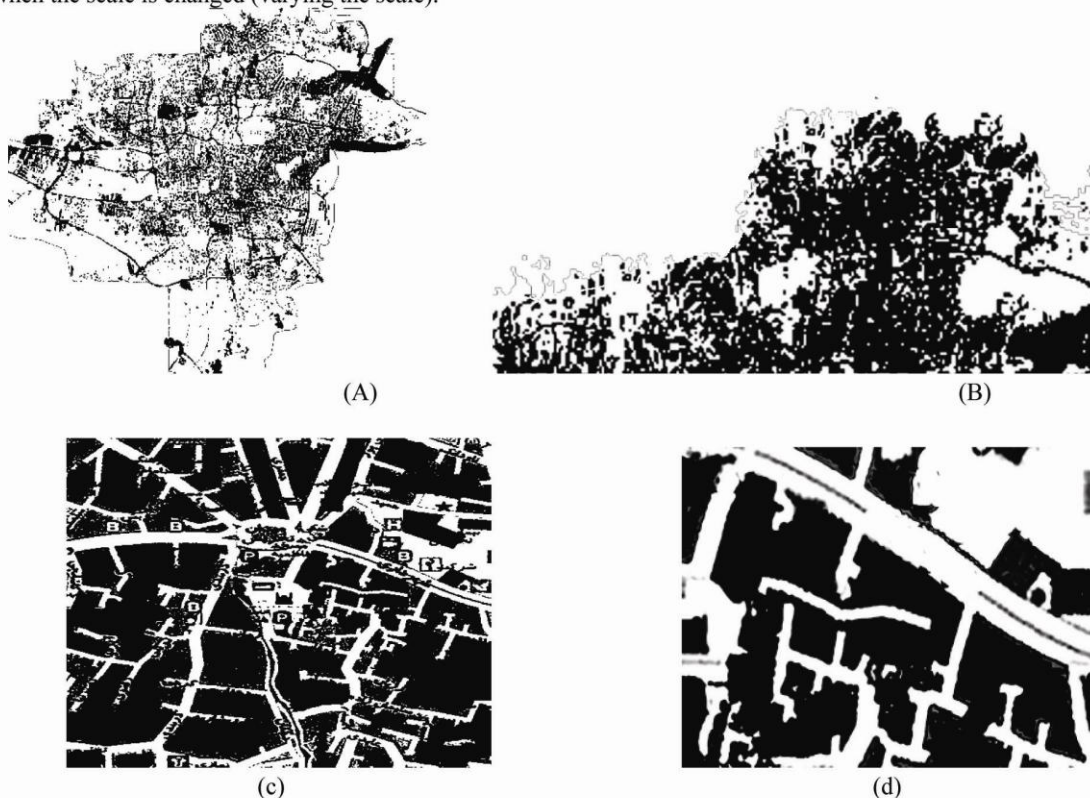


Figure 7:  $F(b)$  measured for Tehran at different scale.

A) Whole Tehran = 1.691      B) Shemiran, the north of Tehran = 1.562



**Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium**  
A Public Forum of Asian Urban Design

C) Tajrish = 1.630 D) one of neighborhoods in Tajrish = 1.806

The following code created by the sequence of fractal measurement, might be considered as a unique mathematical neighborhood ID.

A = 1.691 B = 1.562 C = 1.630 D = 1.806

As the fractal dimensions at different scales vary between 1.00 and 2.00, therefore the decimal numbers are defined as FNID.

A	B	C	D
691	562	630	806

The results for 12 neighborhoods within the case study are summarized in table 4.

Table 4: FNID measured for 12 neighborhoods in Tajrish

Neighborhoods	N1	N2	N3	N4	N5	N6
<b>FNID</b>	691-562-630-806	691-562-630-791	691-562-630-720	691-562-630-601	691-562-630-582	691-562-630-757
Neighborhoods	N7	N8	N9	N10	N11	N12
<b>FNID</b>	691-562-643-672	691-562-643-741	691-562-630-639	691-562-630-803	691-562-630-598	691-562-756-760

As it is obvious in table 4, FNID is contained 12 numbers in 4 sections, A-B-C-D. Respectively each part demonstrates, City ID (A), District ID (B), Local ID (C) and Neighborhood ID (D). As shown in the table in most cases in Tajrish area 9 numbers out of 12 are stable. FNID for N7, N8 and N9 show the change in their local ID, in other words this implies these urban units are not belongs to the same local area as the others. The similarities and the differences in FNID assist us to classify the neighborhoods in a city and draw a new district plan based on morphological urban patterns.

The table also shows Neighborhood ID (D section), changes within a range of 582 to 806, this can be used as a controlling tool, to constrain urban new developments within certain range of fractal dimensions. This will arguably provides an effective way to conserve urban qualities by more accurate method.

#### 4. Conclusion

There is a growing number of evidence to support the application of these theories, However, the evidence exists it is based on either large city wide scale, such as work by M. Batty and P. Longely or very detailed level of individual building design, such as work by Jencks and C. Bovill. The method suggested here, intends to fill the gap by examining the application of the theories at local level of the city (neighborhood scale).

Some outcomes of this method would be:

1. The paper suggests a more appropriate method to analyze and criticize urban forms and spaces in the light of chaos theory and fractal geometry.
2. The process of urban change at macro scale can be more relevant and sensitive by understanding the process of change occurs at micro scale (local level) of a city.
3. Fractal analysis of urban environment indicates a strong relationship between fractal dimension and characteristics of urban pattern by which different urban areas can be distinguished and typified.
4. In some parts of the city, where the fractality (fractal dimension) of their hierarchical structures the same; more visual order and harmonic urban space would be expected.

More research is required to explore the possible relationship between the visual perception of urban forms and their fractal dimensions. However, the same process of fractal assessment of urban patterns introduced in this paper could be

### Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium A Public Forum of Asian Urban Design

---

applied for measuring other urban features such as, skylines, street vistas, street elevations, street edges, etc where a combination of all will introduce the FNID idea as a more effective urban spatial analysis tool.

#### Acknowledgements

I would like to express my gratitude to my PhD supervisors, Professor Tom Muir and Professor Michael Batty for their support and assistance in developing the application of fractal theory in urban morphological studies. This paper has not been possible without the cooperation of the generous technical and scientific support of UCE Research department including Dr Richard Coles. I also appreciate all contributions and theoretical support by Dr Jan Cooper at Oxford Brooks University.

I especially acknowledge all my wife's kind encouragements, Anahita, and her patience during my research.

#### References

- 1 Alexander C, 2000, *The Nature of order*, Oxford University Press, New York.
- 2 Alexander C, 1987, *A New Theory of Urban Design*, Harvard University Press, London.
- 3 Barnsley M, 1993, *Fractals Everywhere*, Academic Press, Cambridge.
- 4 Batty M, 1991, Cities as Fractals: Simulating Growth and Form, in A. Crilly et al (eds.).
- 5 Batty M, Longley P, 1994, *Fractal cities*, Academic Press, London.
- 6 Batty M, Xie Y, 1999, *Self-organized Critically and Urban Development*, Overseas Publisher Association, N. V., Malaysia.
- 7 Batty M, Xie Y, 1996, Preliminary Evidence for a theory of fractal city, *Environment and Planning (A)*, Vol. 28, October, p.p. 1745-1762.
- 8 Batty M, Xie Y, 1994, *From Cells to cities*, Environment and planning B, volume 21, pp 31-48.
- 9 Bovill C, 1996, *Fractal Geometry in Architecture and Design*, Boston
- 10 Blackwell B, 1989, *The Mathematics of chaos*, Oxford.
- 11 Carvalho R et al, 2003, *Scaling and Universality in the Micro-structure of Urban Space*, 4th International Space Syntax Symposium, London.
- 12 Chen Y, et al, 2004, *Multi-fractal Measures of City-size Distributions based on the Three-parameter, Zipf model*, Chaos, Solitons and Fractals vol. 16, UK.
- 13 Cooper J, 2000, *the Potential of Chaos and Fractal Application in Urban Design*, PhD Thesis, Oxford Brooks University, UK.
- 14 Cooper J, 2003, *Fractal assessment of street-level skylines*, Oxford Brooks University, UK.
- 15 Fisher G D, Wagner I, 2003, *Spatial Openness as a Practical Metric For Evaluating Built-up Environment*, Environment and Planning B Vol. 30, pp 37 – 49, London.
- 16 Feder J, 1988, *Fractals*, Plenum Press, New York.
- 17 Glick J. (1987) *Chaos*, New York.
- 18 Gotou H, 2002, *Statistical Correlation between Quantificational Indices and Preference Judgements of Structural Landscapes*, SCIPRESS Publisher, Tokyo.
- 19 Grace R, 1991, *Chaos in Prehistory*, A Seminar at the University of Oslo, USA.
- 20 Hagerhall M et al, 2004, *Fractal dimension of landscape silhouette outlines as a predictor of landscape preference*, Journal of Environmental Psychology, UK.
- 21 Jencks C, 1997, *The Architecture of Jumping Universe*, London/New York.
- 22 Kauffman S, 1993, *Origins of Order, The self-organization and selection in evolution*, New York.
- 23 Kaye H B, 1989, *A Random Walk through Fractal Dimensions*, VCH Publishers, New York.
- 24 Klinger A, Salinger N, 1999, *A Pattern Measure*, Journal of Environmental and Planning (B), San Antonio, USA.
- 25 Lynch K, 1961, *The Image of City*, Mass.: MIT Press, Cambridge.
- 26 Lynch K, 1981, *A Theory of good City Form*, MIT Press, Cambridge.
- 27 Mandelbrot B, 1988, *The Science of Fractal Images*, New York.
- 28 Mandelbrot B, 1982, *The Fractal Geometry of Nature*, New York.
- 29 Matsuba I Et al, 2003, *Scaling Behavior in Urban development Process of Tokyo City and Hierarchical Dynamical Structure*, Chaos, Solitons and Fractals vol. 16, UK.
- 30 Pearson D, 2001, *New Organic Architecture, the Breaking Wave*, University of California Press, Berkeley.
- 31 Roger L, 1993, *Complexity, Life on the Edge of Chaos*, London.
- 32 Russ J C, 1994, *Fractal Surfaces*, Plenum Press, New York.
- 33 Salinger N, 1999, *Complexity and Urban Coherence*, submitted for publication in the Journal of Urban Design, San Antonio, USA.
- 34 Salinger N, 2003, *Connecting the Fractal City*, 5<sup>th</sup> Biennial of town planners in Europe, Barcelona.

**Proceedings of the 3<sup>rd</sup> Great Asian Street Symposium**  
A Public Forum of Asian Urban Design

---

- 35 Stamps A, 2002, *Fractals, Skylines, Nature and Beauty*, Landscape and Urban Planning Vol. 60, UK.
- 36 Steadman P, March L, 1974, *The Geometry of Environment*, Alger Press Limited, Canada.
- 37 Wioldlich W, 1994, *Chaos, Solitons and fractals*, in 'Settlement formation at the meso scale', Vol 4. No. 4 pp 507-583.
- 38 Wallczek, J, 2000, *Self Organized Biological Dynamics & Nonlinear Control*, Cambridge University Press, UK.
- 39 Webster C J, 1995, *Urban morphological fingerprints*, Environment planning and design, volume 22, pp 279-297.